



## A STUDY ON REMANUFACTURING OF USED PRODUCTS IN A VENDOR-BUYER SUPPLY CHAIN

**W. Ritha and J. C. Eveline**

Department of Mathematics, Holy Cross College (Autonomous), Tiruchirappalli, India.  
E-mail : ritha\_prakash@yahoo.co.in

---

### ABSTRACT :

*Protecting the environment has become a priority for most countries in recent years. Recycling material and remanufacturing used products are inevitable options to reduce waste generation and the exploitation of natural resources. Remanufacturing is often considered as a environmental preferable choice of end of life option in comparison to material recycling or manufacturing of new products. The forward supply chain essentially involves the movement of products from upstream suppliers to the downstream customers while the reverse supply chain involves the movement of used products from customers to upstream suppliers. This paper proposes the study of remanufacturing in the closed loop supply chain consisting of a vendor(manufacturer) and a buyer(retailer).The used products collected by the buyer from the customers are remanufactured by the vendor. The inventory holding cost of collected used products are involved in the model. The optimal lot size of remanufactured products and the collection rate of used products to be remanufactured are obtained which minimize the joint total cost of the supply chain. Finally a numerical example is provided for the described model.*

**Keywords :** *Remanufacturing, EOQ, Vendor-Buyer, Used products, Environment, Reverse logistics.*

**Keywords :** *Annual maximum, annual minimum, ambient air temperature, Dhubri, analytical method of determination.*

---

### 1. INTRODUCTION

Product recovery (repair, refurbishing, remanufacturing) is receiving increasing attention. In the past, engagement in recovery activities was often driven by legislation or by associated environmentally friendly image. But nowadays the main reason for companies to become involved with product recovery is economical. Being active in product recovery reduces the need for virgin materials and thus leads to reduced costs. Recoverable manufacturing systems minimize the environmental impact of industry by reusing materials and reducing energy use. In such systems that are environmentally conscious, products are returned from end users and travel back in the reverse supply chain.

To manufacturers, once a product has been returned to a company, it has several options from which to choose. The first option is to sell the product as a used product if it can be established that it meets sufficient quality levels. The second option is to clean and repair the product to working order. Product repair involves fixing and replacement of failed parts. The third option is to sell the product as a refurbished unit. The product does not lose its identity and is brought back to a specified quality level. The fourth option is to remanufacture. In this option the product will undergo the reverse channel at the fabrication stage where it would be disassembled, remanufactured and reassembled to flow back through the retail outlet back to the consumers as a remanufactured product. The fifth

option is to retrieve one or more valuable parts from the product. The sixth option is to recycle. The main purpose of recycling is reuse materials from used components and products. The seventh option is to recover the energy put in the product through incineration. The last option is disposal. The general goal for any value channel is to keep all materials within the channel and thus minimize any flow into the external environment. The basis of recoverable manufacturing system is remanufacturing. Remanufacturing offers several advantages as a form of waste reduction since it is profitable and environmentally conscious.

Supply chain management has received tremendous attention both from the business world and from academic researchers. Closed loop supply chain consists of both forward supply chain and a reverse supply chain. A forward supply chain is a combination of processes to fulfill customer's requests and includes all possible entities like suppliers, manufacturers, transporters, ware houses, retailers and customers. The management of the reverse flows is an extension of the traditional supply chains with used products or material either returning to reprocessing organizations or being discarded. Reverse supply chain management is defined as the effective and efficient management of the series of activities required to retrieve a product from a customer and either dispose of it or recover value.

The remainder of the paper is organized as follows: Section 2 describes the relevant literature. Section 3 presents the notations and assumptions. The formulation of the model is provided in Section 4. Section 5 illustrates a numerical example. The paper concludes in Section 6. A list of references is also provided.

## 2. LITERATURE REVIEW

The importance of the repairable/recoverable inventory problem was recognized back in the 1960's. Schradly (1967) determined the optimal procurement and repair quantities for the repairable inventory system of an EOQ model. Mabini et al. (1992) studied the stock out as service level for the repairable system and besides extended to the case of multiple products with limited repair capacity. Fleischmann et al. (2000) explored the design of logistics networks and established general characteristics of product recovery systems. Koh et al. (2002) enquired a joint EOQ and EPQ model to optimally determine EOQ for procurement and inventory level of recoverable products concurrently. Teunter (2004) acquired simple square root formula to determine the optimal production and recovery batch quantities for two classes of policies: (1,R) and (P,1). Inderfurth et al. (2005) included the deteriorating nature of reworkable products into an EPQ-based recoverable system and found the optimal production lot size. A lot of researches addressed the issues of repair and disposal of used products simultaneously. The optimal setup numbers for production and repairs in a collection time interval at a fixed waste disposal rate were derived by Richter (1996a) and assumed waste disposal rate as a decision variable. Richter (1996b) further observed the behaviours of EOQ-related cost factors and/or additional non-EOQ-related cost factors of the repairable system. The EOQ repair and waste disposal problem with integer setup numbers was studied by Richter and Dobos (1999) and showed that the pure strategy for either total repair or total waste disposal is dominant. Teunter (2001) evaluated the recoverable item inventory problem with disposal consideration by involving different holding costs for manufactured and recovered products. Jaber et al. (2014) coped with economic order quantity models where the imperfect items are either sent to an independent repair shop or replaced by good ones from a local supplier. There is growing number of researches addressing remanufacturing issues in a closed-loop supply chain. Guide and Van Wassenhove (2001) showed that the acquisition of used products for remanufacturing is profitable. Heese et al. (2005) developed a quantitative model to investigate the consequences of used products take-back on firms, industry and customers and suggested that a manufacturer can increase both profit margins and sales.

Savaskan et al. (2004) studied four different channel structures in closed-loop supply chains with product remanufacturing and compared these models with respect to return rates, retail prices and channel members' profits. A deterministic mixed integer linear programming model with the network design problem for a closed loop supply chain under uncertainty was developed by Pishvae et al. (2011). Hong and Yeh (2012) concluded that the retailer collection model is better off when the third-party collector is a non-profit organization. Inventory management of produced/remanufactured/repaired and returned items has been receiving increasing attention in recent years. Richter and Weber (2001) extended the Wagner/Whitin model to consider additional variable manufacturing and remanufacturing costs and explored the impact of the disposal excess inventory on the solution. Jayaraman (2006) provided an analytical model for closed-loop supply chains with product recovery and reuse to aid operational decision makers for production planning and control. Chung et al. (2008) developed a closed loop supply chain model with remanufacturing and maximized the joint profits of the supplier, the manufacturer, the third party collector and the retailer. Saadany and Jaber (2008) studied about the coordination of two-level supply chain where the production interruptions are permitted to restore process quality whenever the production process shifts to the out-of-control state. Saadany and Jaber (2010) created and analyzed productions, remanufacture and waste disposal EPQ models. An extended joint economic lot size problem in which the return flow of repairable (remanufacturable) used products was incorporated by Dobos et al. (2011) where the returned products are remanufactured by the vendor.

This paper is an extension of “ **Optimal replenishment quantity of new products and return rate of used products for a retailer** ” by Chih-Chung Lo, Cheng-Kang Chen and Tzu-Chun Weng. In this paper the vendor is engaged in remanufacturing of used products collected by the buyer from the customers. The inventory holding cost for the used products is also included in the model.

### 3. NOTATIONS AND ASSUMPTIONS

#### Notations

$D$	demand of the buyer per time unit,
$P_M$	manufacturing productivity of the vendor, $P_M > D$ ,
$P_R$	remanufacturing productivity of the vendor, $P_R > D$ ,
$K_b$	setup cost of an ordering of the buyer,
$h_b$	holding costs of the new products of the buyer,
$u_b$	holding costs of the used products of the buyer, $h_b > u_b$ ,
$d_b$	disposal costs of the used products of the buyer,
$K_v$	setup cost of an ordering of the vendor,
$h_v$	holding costs of the new products of the vendor,
$u_v$	holding costs of the used products of the vendor, $h_v > u_v$ ,
$C_v$	unit purchasing cost of the product of vendor,
$Q$	lot size of remanufactured products of the system, (decision variable)
$\tau$	return rate of used items to be collected and remanufactured, (decision variable) ( $0 < \tau < 1$ )
$A$	The unit cost of collecting, holding and handling a returned product which covers the collecting fee paid by the retailer to consumers.
$b$	The unit price of a collected used products sold by the retailer to the manufacturer, the salvage value of collected used products $b - A$ is supposed to be positive.

$I(\tau)$  The investment cost of the retailer in collecting used product activities, which is assumed to be a function of return rate  $\tau$ .

$h_{MR}(\tau)$  holding cost coefficient for manufacturing,

$h_{RM}(\tau)$  holding cost coefficient for remanufacturing,

**Assumptions**

The assumptions of this model are

1. Single product case.
2. Instantaneous replenishment of the product.
3. Shortage is not permitted.
4. Demand rate is constant and deterministic.
5. Infinite planning horizon.
6.  $I(\tau) = C_L\tau^2$  denotes the investment cost function in collecting used products.
7. The cumulative return rate of used products at the current replenishment cycle is expressed by a geometric series as  $\sum_{i=1}^{\infty} \tau r^{i-1} = \tau / (1 - r)$ , where  $\tau$  is the initial value and common ratio  $r$ ,  $0 \leq r \leq 1$ . It is noted that the total number of collected used products should be non negative and cannot exceed the number of products sold at each replenishment cycle. Hence the constraint  $0 \leq \frac{\tau}{1-r} \leq 1$  holds.
8. The vendor collects the used products from the buyer to remanufacture.

**4. MODEL FORMULATION**

Consider a supply chain consisting of a vendor and a buyer. The buyer is assumed not only to sell the products to public consumers but also to collect those sold used products from them. The collected used products are sent to the vendor for remanufacturing process. The used products sent by the buyer in batches was remanufactured by the vendor.

The joint total cost of the system per cycle is,

$$JTCPC(Q, \tau) = (K_b + K_v) + \frac{Q^2}{2D} [h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau)] + C_v Q + C_L \tau^2 - \frac{(b - A)\tau Q}{1 - r} + d_b(1 - \tau)Q$$

where

$$h_{MR}(\tau) = \tau^2 \cdot \left[ h_v \cdot \left( \frac{D}{P_M} - \frac{D}{P_R} \right) - u_v \frac{D}{P_R} \right] - 2\tau \cdot \left[ h_v \cdot \left( \frac{D}{P_M} - \frac{D}{P_R} \right) - u_v \right] + h_v \frac{D}{P_M},$$

$$h_{RM}(\tau) = \tau^2 \cdot \left[ (h_v - u_v) \cdot \left( \frac{D}{P_R} - \frac{D}{P_M} \right) + u_v \frac{D}{P_M} \right] + 2\tau u_v \cdot \left( 1 - \frac{D}{P_M} \right) + h_v \frac{D}{P_M}.$$

The corresponding joint total cost per unit time ( $JTCPUT$ ) can be obtained by dividing the joint total cost per cycle ( $JTCPC$ ) by the cycle length  $\frac{Q}{D}$ . The objective of the model is to minimize the total cost per unit time, subject to the return rate constraint  $0 \leq \frac{\tau}{1-r} \leq 1$ . Namely,

Minimize

$$JTCPUT(Q, \tau) = (K_b + K_v) \frac{D}{Q} + \frac{Q}{2} [h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau)] + C_v D + \frac{C_L D \tau^2}{Q} - (b - A) D \frac{\tau}{1-r} + d_b(1 - \tau) D \tag{1}$$

Subject to:  $0 \leq \frac{\tau}{1-r} \leq 1$  (2)

In order to solve the proposed non linear programming problem shown in (1), the constraint  $0 \leq \frac{\tau}{1-r} \leq 1$  is ignored and the partial derivatives of  $JTCPUT(Q, \tau)$  with respect to  $Q$  and  $\tau$  are obtained to find the optimal values.

To find the optimal value of  $Q$  for fixed value of  $\tau$ , the first partial derivative of  $JTCPUT(Q, \tau)$  is set to zero and the optimal value of  $Q$  is given by,

$$Q^*(\tau) = \frac{\sqrt{2D(K_b + K_v + C_L\tau^2)}}{\sqrt{(h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau))}} \tag{3}$$

Substituting (3) in (1), the  $JTCPUT(Q, \tau)$  is expressed as,

$$JTCPUT(\tau) = \sqrt{2D(K_b + K_v + C_L\tau^2)}(h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau)) + C_v D - (b - A)D \frac{\tau}{1 - r} + d_b(1\tau)D \tag{4}$$

Differentiating partially  $JTCPUT(\tau)$  with respect to  $\tau$  and equating to zero, gives the optimal solution of  $\tau$ .

$$\frac{\partial JTCPUT}{\partial \tau} = \sqrt{2D} \left\{ \frac{\sqrt{(K_b + K_v + C_L\tau^2)}}{2\sqrt{(h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau))}} (u_b + h'_{MR}(\tau) + h'_{RM}(\tau)) + \frac{\sqrt{(h_b + \tau u_b + h_{MR}(\tau) + h_{RM}(\tau))}}{\sqrt{(K_b + K_v + C_L\tau^2)}} (C_L\tau) \right\} - \frac{(b - A)D}{1 - r} - d_b D = 0 \tag{5}$$

### 5. NUMERICAL EXAMPLE

In this section we establish a numerical example for the above proposed model. The following parameters are used for finding the solution:

$D = 1,000$  piece/year,  $P_M = 2,500$  piece/year,  $P_R = 1,200$  piece/year,  $K_b = 100$  \$/ordering,  $h_b = 5$  \$/piece/year,  $u_b = 1$  \$/piece/year,  $K_v = 1,000$  \$/ordering,  $h_v = 3$  \$/piece/year,  $u_v = 1$  \$/piece/year,  $d_b = 1$  \$/piece,  $C_v = 10$ ,  $b = 2$ ,  $A = 1$ ,  $C_L = 50000$  and  $r = 0.1$ .

From (5) we found  $\tau^* = 0.00141492$

From (3) we found  $Q^* = 544.921$  and

From (1) we get  $JTCPUT^* = 15,034.661$

### 6. CONCLUSION

This paper studies the remanufacturing of used products in the supply chain comprising of a vendor and a buyer. Remanufacturing is an eco-friendly option as it uses less energy than manufacturing a new product, reduces CO<sub>2</sub> emissions, reduces flow of material to landfill and reduces raw material consumption. The incorporation of remanufacturing in the supply chain reduces the total cost of the system.

### REFERENCES

1. Chih-Chung Lo., Cheng-Kang Chen , and Tzu-Chun Weng.(2016). “Optimal replenishment quantity of new products for a retailer.” Applied Mathematical Modelling 40(23-24): 9754-9766.
2. Chung , S.L., H.M. Wee, and P.C. Yang .(2008). “Optimal policy for a closed-loop supply chain inventory system with remanufacturing”, Math. Comput. Model 48: 867–881.

3. Dobos, I., Gobsch, B., Pakhomova, N., Richter, K.: Remanufacturing of used products in a closed-loop supply chain. In: Csutora, M., Kerekes, S. (eds.) *Accounting for climate change– What and how to measure*, pp. 130–146. Proceedings of the EMAN-EU 2011 Conference, 24–25 January 2011, Budapest, Hungary (2011).
4. El Saadany, A., and M.Y.Jaber.(2008). “Coordinating a two-level supply chain with production interruptions to restore process quality.” *Comput. Ind. Eng* 54: 95–109.
5. El Saadany , A., and M. Y. Jaber.(2010). “A production/remanufacturing inventory model with price and quality dependent return rate.” *Comput. Ind. Eng* 58: 352–362.
6. Fleischmann, M., H.R. Krikke, R. Dekker, and S.D.P. Flapper.(2000). “A characterization of logistics networks for product recovery.” *Omega* 28 : 653–666.
7. Guide, V.D.R., and L.N. Van Wassenhove.(2001). “Managing product returns for remanufacturing.” *Prod. Oper. Manag* 10:142–155.
8. Heese, H.S., K. Cattani, G. Ferrer, W. Gilland, and A.V. Roth.(2005). “Competitive advantage through take-back of used products.” *Eur. J. Oper. Res* 164: 143–157.
9. Hong, I.H., and J.S. Yeh.(2012). “Modeling closed-loop supply chains in the electronics industry : A retailer collection application.” *Transport. Res. E Logist. Transport. Rev* 48: 817–829.